Measurement of $|V_{ub}|$ in semi-inclusive charmless $B \to \pi X$ decays

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We study semi-inclusive charmless decays $B \to \pi X$, where X does not contain a charm (anti)quark. The mode $\bar{B}^0 \to \pi^- X$ turns out to be be particularly useful for determination of the CKM matrix element $|V_{ub}|$. We present the branching ratio (BR) of $\bar{B}^0 \to \pi^- X$ as a function of $|V_{ub}|$, with an estimation of possible uncertainty. The BR is expected to be an order of 10^{-4}.

1. Introduction

A precise measurement of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [1] is one of the key issues in the study of B mesons and B factory experiments. In particular, the accurate determination of V_{ub} is one of the most challenging problems in B physics.

Theoretical and experimental studies for probing V_{ub} have been mostly focused on the semileptonic B meson decays. The CLEO result obtained using the exclusive semileptonic decay $B \to \rho l \bar{\nu}$ [2] : (in 10^{-3})

$$|V_{ub}| = (3.25 \pm 0.14^{+0.21}_{-0.29} \pm 0.55(\text{model})). \quad (1)$$

The OPAL data obtained using the inclusive decay $B \to X_u l \bar{\nu}$ [3] : (in 10^{-3})

$$|V_{ub}| = (4.00 \pm 0.65^{+0.67}_{-0.76} \pm 0.19(\text{HQE})). \quad (2)$$

In this work [4] we study semi-inclusive charmless decays $B \to \pi X$ and investigate the possibility of extracting $|V_{ub}|$ from these processes. Compared to the exclusive decays, these semi-inclusive decays are generally expected to have less hadronic uncertainty and larger branching ratios. There are several possible processes in $B \to \pi X$ type decays, such as $\bar{B}^0 \to \pi^{\pm(0)} X$, $B^0 \to \pi^{\pm(0)} X$, $B^\pm \to \pi^{\pm(0)} X$, where X does not contain a charm (anti)quark. Among these processes of the type $B \to \pi X$, we identify a certain mode, $\bar{B}^0 \to \pi^- X$, whose analysis is theoretically clean and which can be used for determining $|V_{ub}|$. Then, we calculate the branching ratio (BR) of $\bar{B}^0 \to \pi^- X$, and present the result as a function of $|V_{ub}|$ with an estimation of possible uncertainty. We also consider the $B^0 - \bar{B}^0$ mixing effect through $\bar{B}^0 \to B^0 \to \pi^- X$.

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2. Classification of semi-inclusive charmless $B \to \pi X$ decays

Among the semi-inclusive charmless $B \to \pi X$ decays, let us first consider the mode $\bar{B}^0 \to \pi^- X$. Contributions for the decay amplitude of this mode arise from the color-favored tree ($b \to u\bar{u}d$) diagram and the $b \to d$ penguin diagram, so that the tree diagram contribution dominates. The charged pion π^- in the final state can be produced via a W boson emission at tree level and is expected to be energetic ($\sim m_B/2$). The decay amplitude can be written as

$$\begin{aligned} & A(\bar{B}^0 \to \pi^- X) \\ = \; & A(b \to \pi^- u) \cdot h(u\bar{d} \to X(u\bar{d})) \, , \end{aligned} \quad (3)$$

where h denotes a hadronization function describing the combination of the $u\bar{d}$ pair to make the final state X. To obtain the decay rate, $X(u\bar{d})$ should be summed over all the possible states, such as $\pi^+\pi^0$, $\pi\pi\pi$ etc, so this process is effectively a *two*-body decay process of $b \to \pi^- u$. Thus, in this mode, no hadronic form factors are involved, and as a result the model-dependence and uncertainty relevant to hadronic form factors do not appear. We note that the *energetic* charged pion π^- in the final state can be a characteristic signal for this mode. (The net electric charge of X should be *positive* so that π^- cannot be produced in the case of $X = \pi\pi$.)

Now let us consider the mode $B^- \to \pi^- X$. Various contributions are responsible for this process : the color-favored tree diagram, the color-suppressed tree diagram, the $b \to d$ and $b \to s$ penguin diagrams. The color-favored tree contribution and one of $b \to d$ penguin contributions are similar to those in $\bar{B}^0 \to \pi^- X$, which are effectively two-body type ($b \to \pi^- u$) processes. But, the color-suppressed tree and other penguins differ from those in $\bar{B}^0 \to \pi^- X$. In fact, these diagrams correspond to a three-body decay process of $B^- \to \pi^- u\bar{u}$ in the parton model approximation. The charged pion π^- in the final state contains the spectator antiquark $\bar{u}$. So the analysis involves the hadronic form factor for the $B \to \pi$ transition which is model-dependent. Furthermore, the $b \to s$ penguin contribution is not suppressed compared to the tree contributions, but dominant in this mode. Therefore, compared to the case of $\bar{B}^0 \to \pi^- X$, the analysis of this mode is much more complicated and involves larger uncertainty.

Other modes of the type $B \to \pi X$ can be similarly classified. For instance, in the mode $B^0 \to \pi^- X$, the color-favored tree ($\bar{b} \to \bar{u}u\bar{d}$ and $\bar{b} \to \bar{u}u\bar{s}$) diagrams and the $b \to d$ and $b \to s$ penguin diagrams are responsible for the decay process. In this case, the charged pion π^- contains the spectator quark d so that the process is effectively a three-body decay $B^0 \to \pi^- u\bar{d}\,(\bar{s})$ and the hadronic form factor for the $B \to \pi$ transition is involved. Other processes are effectively a combination of the two-body decay process ($b \to \pi q$) and the three-body decay process ($B^- \to \pi^- q\bar{q}'$).

3. Analysis of $\bar{B}^0 \to \pi^- X$ decay

We have seen that the process $\bar{B}^0 \to \pi^- X$ is particularly interesting, because it is effectively the two-body decay process $b \to \pi^- u$ and no uncertainty from hadronic form factors is involved. Thus, its theoretical analysis is expected to be quite clean.

We calculate the BR of the process $\bar{B}^0 \to \pi^- X$, where X can contain an up quark and a down antiquark. We use the effective Hamiltonian and the effective Wilson coefficients given in Ref. [5]. The BR can be expressed as a polynomial of $|V_{ub}|$:

$$\mathcal{B}(\bar{B}^0 \to \pi^- X) = \left|\frac{V_{ub}}{0.004}\right|^2 \cdot \mathcal{B}_2 + \left|\frac{V_{ub}}{0.004}\right| \cdot \mathcal{B}_1 + \mathcal{B}_0 \ , \qquad (4)$$

where for convenience we have scaled $|V_{ub}|$ by the factor 0.004 (the central value of the OPAL data).

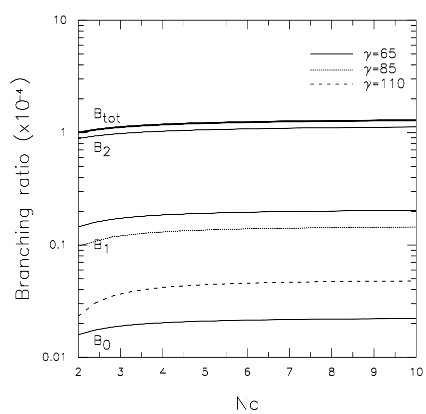

Figure 1. The branching ratio (in 10^{-4}) versus N_c for $\bar{B}^0 \to \pi^- X$ decay. $B_{tot}(\equiv \mathcal{B})$ has been calculated using $|V_{ub}| = 0.004$ and is denoted by the bold solid line. The solid, dotted, and dashed lines correspond to $\gamma = 65^0,\ 85^0,\ 110^0$, respectively.

In Figure 1, we present the BR of $\bar{B}^0 \to \pi^- X$

as a function of the effective number of color N_c for three different values of the CP phase angle $\gamma(\equiv \phi_3) = 65^0,\ 85^0,\ 110^0$. $\mathcal{B}_2$ and $\mathcal{B}_0$ are independent of $\gamma(\equiv \phi_3)$, and only $\mathcal{B}_1$ depends on $\gamma(\equiv \phi_3)$. Three different lines for $\mathcal{B}_1$ correspond to the relevant values of $\gamma(\equiv \phi_3)$, respectively. It is clearly shown that $\mathcal{B}_2$ is dominant. An reperesentative value of $\mathcal{B}$ for $|V_{ub}| = 0.004$ and $\gamma(\equiv \phi_3) = 85^0$ is shown as the bold solid line in the figure. The value of $\mathcal{B}$ does not vary much as N_c varies.

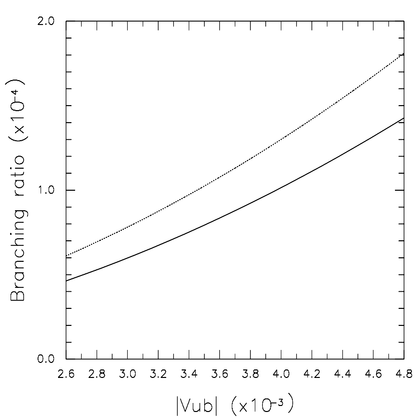

Figure 2. The branching ratio (in 10^{-4}) versus $|V_{ub}|$ for $\bar{B}^0 \to \pi^- X$ decay. The solid and the dotted line correspond to the smallest and the largest value of $\mathcal{B}$ in the given parameter space, respectively.

In Figure 2, the BR of $\bar{B}^0 \to \pi^- X$ is presented as a function of $|V_{ub}|$. We vary the value of N_c and $\gamma(\equiv \phi_3)$ in a reasonable range: from $N_c = 2$ to 10, and from $\gamma(\equiv \phi_3) = 60^0$ to 110^0. The solid and the dotted line correspond to the smallest and the largest value of $\mathcal{B}$ in the given parameter space, respectively. The BR is an order of 10^{-4}. For the given $|V_{ub}|$, the BR $\mathcal{B}$ is estimated with a relatively small error ($< 15\%$). Reversely, for the given (i.e., experimentally measured) BR $\mathcal{B}$, the value of $|V_{ub}|$ can be determined with a small error ($< 10\%$). (Of course, since in a practical experiment the BR would be measured with some errors, $|V_{ub}|$ could be determined with larger error: e.g., for $\mathcal{B} = (1.0 \pm 0.1) \times 10^{-4}$, our result expects $|V_{ub}| = (3.7 \pm 0.47) \times 10^{-3}$.)

Using the decay process $\bar{B}^0 \to \pi^- X$, one may need to consider the $B^0 - \bar{B}^0$ mixing effect: $\bar{B}^0 \to B^0 \to \pi^- X$. The neutral $\bar{B}^0$ has about 18% probability of decaying as the opposite flavor B^0 [6]. It turns out [4] that even considering the effect from the $B^0 - \bar{B}^0$ mixing, our result holds with resonable accuracy.

4. Conclusion

We have shown that among semi-inclusive charmless $B \to \pi X$ decays, the process $\bar{B}^0 \to \pi^- X$ is particularly interesting and one can determine $|V_{ub}|$ with reasonable accuracy, by measuring the BR of $\bar{B}^0 \to \pi^- X$.

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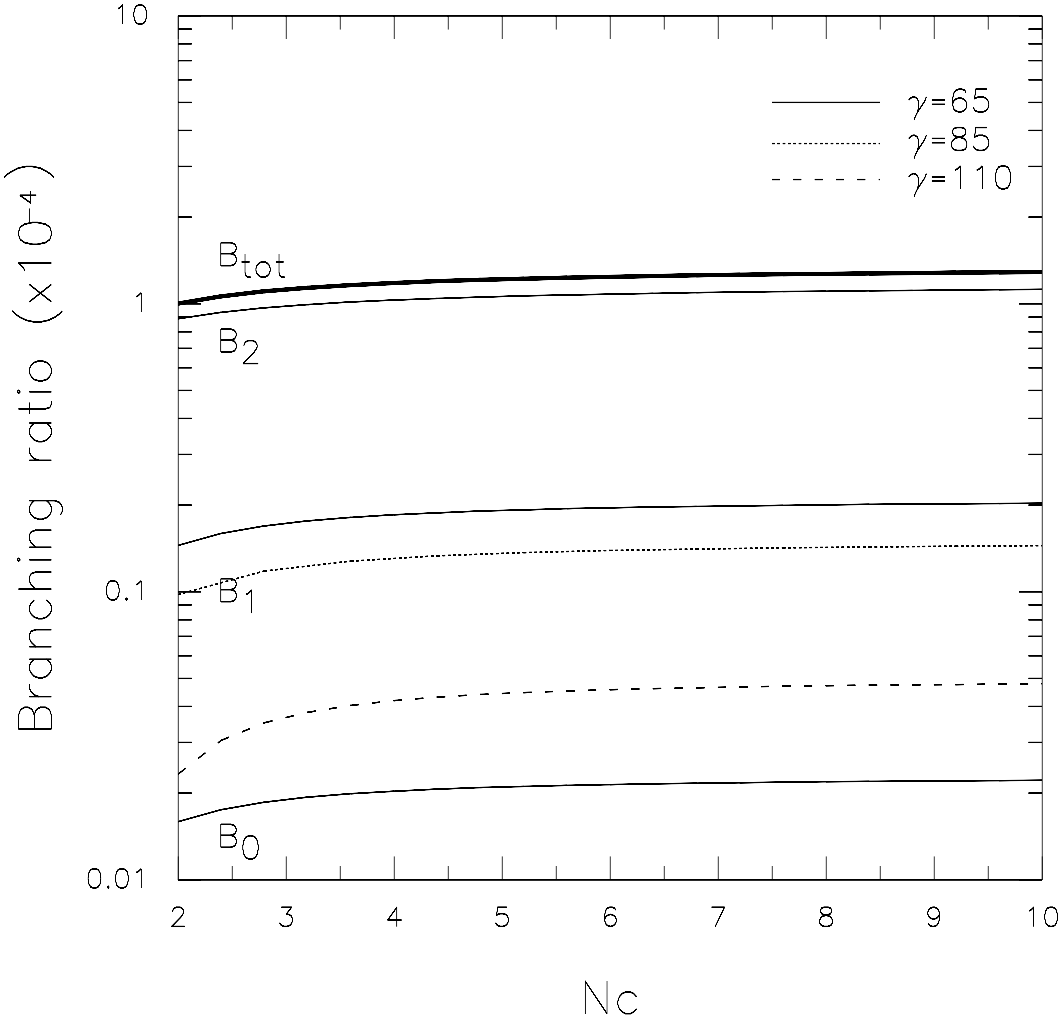

10
1
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Nc
Branching ratio (x10⁻⁴)
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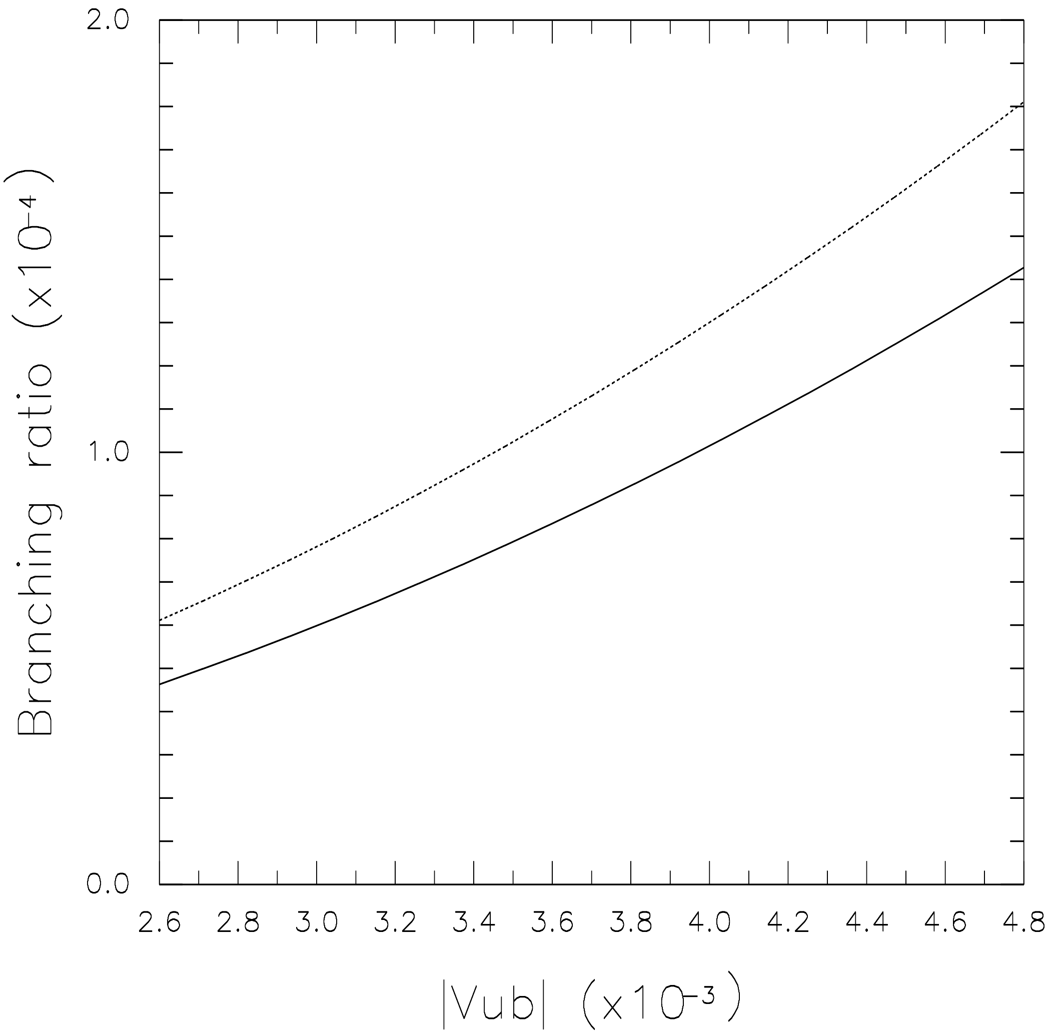
γ=85
γ=110
B_tot
B_2
B_1
B_0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Branching ratio (x10^{-4})
|Vub| (x10^{-3})